HYPOTHESES AND AGREEMENT. FOR A PRAGMATIC AND DYNAMIC APPROACH TO HYPOTHESES AS CONDITIONAL SYNTHESES

HIPÓTESIS Y ACUERDO. POR UN ENFOQUE PRAGMÁTICO Y DINÁMICO DE LAS HIPÓTESIS COMO SÍNTESIS CONDICIONALES

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ABSTRACT
This paper aims to contribute to the understanding of hypotheses in science from a dynamic and pragmatic approach. In this sense, we argue that the notion of hypothesis corresponds to the performative act of generating a conditional relation between elements of knowledge. For this purpose, we appeal to the notion of demonstration by hypothesis that Aristotle outlines in his Prior Analytics. On the latter, Aristoteles outlines the notions of agreement, substitution, and relation between two demonstrations. Thus, we circumscribe the generation of hypotheses to the field of dialectical interactions in which human beings and their actions as argumentative agents with purposes and ends play a central role. And at the same time, we distance ourselves from the static identification between hypotheses and propositions.

Keywords: hypotheses / logic / conditionals / demonstration / inferential relation

RESUMEN
Este artículo pretende contribuir a la comprensión de las hipótesis en ciencia desde un enfoque dinámico y pragmático. En este sentido, argumentamos que la noción de hipótesis corresponde al acto performativo de generar una relación condicional entre elementos de conocimiento. Para ello, apelamos a la noción de demostración por hipótesis que Aristóteles esboza en sus Primeros Analíticos. En torno a este tipo de demostración, Aristóteles esboza las nociones de concordancia, sustitución y relación entre dos demostraciones. De este modo, circunscribimos la generación de hipótesis al ámbito de las interacciones dialécticas en las que los seres humanos y sus acciones como agentes argumentativos con propósitos y fines desempeñan un papel central. Y, al mismo tiempo, nos distanciamos de la identificación estática entre hipótesis y proposiciones.

Palabras clave: hipótesis / lógica / condicionales / demostración / relación inferencial

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1. Hypotheses and inferences

What is a hypothesis, and what place does it occupy in an argument? It is challenging to give a specific and complete answer to this question. Following the general literature on History of Philosophy and Mathematics (Copleston, 1958; Heath, 1921; Barnes, 1969; Eves, 1969; among others), the notion of hypothesis accompanies us from the very origins of philosophy in the West, in the pre-Socratic thought, and to the present day. However, it seems not to have received all the attention it deserves (as we already pointed out in Redmond 2022 and Redmond & López-Orellana 2022, 2023a). For example, there is currently no bibliographic material that systematically traces this historical and conceptual journey with all its particularities. And where we focus our attention on philosophy, it's hard not to meet it.

The first traces of the notion of hypothesis can be found in Greek geometricians, from 600 B.C. onwards. Plato recovers this notion from the geometricians in his Dialogues (*Menon* 86b-e). Then Aristotle takes it up again and develops a specific sense of it: reductions to the impossible (*reductio ad impossibile*). Indeed, Aristotle recognized in his *Prior Analytics* (50b) the importance of this type of proof and promised a corresponding theoretical development. Still, there is no record that he has done it.[1] As we know, almost all of the early geometers' writings have been lost. We only have Euclid's texts (400 years later) as a systematization of ancient geometry knowledge, which survived the oblivion of time. But the evidence we find in the Elements differs from its predecessors. Throughout this period, including Aristotle, the notion of hypothesis presents a polysemy that is difficult to determine. For the modest purpose of this paper, we only focus on its use in logical demonstrations. So, it is necessary to point out that by 'demonstration,' we refer to this procedure that did not appear before the Greek geometers: "It should be noted, however, that one cannot find in all ancient oriental mathematics even a single instance of what we today call a demonstration" (Eves, 1969, p. 28).

To delimit the hypothesis notion in terms of its use —in the logical proofs mentioned above, where we set up our reflection, we resort to the distinctions that André Lalande (1997) presents in his dictionary, one of the best articles —according to our criteria— on what a hypothesis is. Lalande offers three divisions that, we believe, encompass in a general way the most prominent connotations of this notion. Lalande (1997, p. 428, our translation) outlines it as follows:

1 "Many other conclusions also are reached by hypothesis, and these require further study and clear explanation. What their differences are, and in how many ways a hypothetical conclusion is effected, will be described later." (Aristotle, 1962, p. 389).

A. Mathematical sense: *data of a problem*, starting points to demonstrate.
B. Proposition received without knowing if it is V or F and from which others are inferred to contrast with experience.
C. A dubious but plausible conjecture by which imagination anticipates knowledge and which is *destined to be evaluated* (i) by direct observation or (ii) by all its consequences with observation (anticipated interpretation of phenomena).

From our view, sense A points to the logical sense that interests us, while B and C point to its use within experimental sciences (which, of course, is not entirely disconnected from its logical sense). We consider the latter as a demarcation criterion to identify what we call their *inferential use* of them. Indeed, senses B and C, which we understand as non-inferential, point to using the notion of hypothesis in scientific practice and, more appropriately, in experimental practice involving measurements, empirical evaluations, contrasts, etc. It should be noted that, in none of the three cases, the origin of the hypothesis is indicated, whether it is the conclusion of reasoning or if it comes from the spontaneous creativity of an agent. Furthermore, according to how Lalande presents it, we could express the three senses as a conditional relation where the hypothesis occupies the place of the antecedent. The latter is indeed what we are questioning in our paper.

A consequence of the above is that, while in B and C, they may lose their character as a hypothesis after a direct evaluation (not in the case of the indirect one), in A, it would maintain that character throughout the process. Regarding sense A, Plato states (*Rep.* 510c) the following:

> Let's try again. You see, you will understand it more easily after this explanation. I think you know that students of geometry, calculation, and the like hypothesize the odd and the even, the various figures, the three kinds of angles, and other things akin to these in each of their investigations, regarding them as known. These they treat as hypotheses and do not think it necessary to give any argument for them, either to themselves or to others, as if they were evident to everyone. And going from these first principles through the remaining steps, they arrive in full agreement at the point they set out to reach in their investigation. (Plato, 2005, p. 206)

This sense of hypothesis was already recognized and used by different commentators of the work of the Greek philosophers and geometricians. For example, according to Barnes (2007, p. 90), we have Philoponus' comments to the *Prior Analytics* (*APr.* 243.15-24) and Proclus who in his comments to

Euclid's work states that "'All isosceles triangles have their base-angles equal', you 'suppose' you have an isosceles triangle and 'conclude' to the equality of its base-angles" (*Eucl.* 252.5-23). In short, as Barnes states, "in a conditional, you 'lay down' the antecedent and see what follows" (2007, p. 90).

In short, we want to point out that we will focus on the meaning of hypotheses in logical demonstrations. Not so much on their role as statements intended to be corroborated or not in experience.

2. Hypothesis and indeterminacy

Both in the mathematical meaning —or inferential, as we call it— and in the other senses closer to scientific practice, using hypotheses seems to introduce a certain indeterminacy into these procedures. In C, indeterminacy comes from the dubious but plausible conjecture introduced into the cognitive process to be tested. Something similar happens with sense B of the hypothesis, where it is quite possible that the 'truthfully indeterminate statement introduced' will never change its status. But both cases are still standing because of the cognitive success of their consequences (as can be understood, for example, in the case of Semmelweiss and his hypothesis of cadaveric matter). This is the sense in which Newton criticizes.[2] Indeed, before Newton, skeptics had already been confronted critically with the use of hypotheses in the production of knowledge. For this last, it is enough to focus on the texts of the Sextus Empiricus to verify the central place occupied by the notion of hypothesis in his criticisms (Barnes, 2007, pp. 95ff).

On the contrary, from our point of view, there seems to be no indeterminacy in the case of A. Those elements (even objects) from which a demonstration is initiated are not veritatively indeterminate: neither V nor F. We start from previous theorems or suitable objects (not including the evidence or the justifications for them) and then continue with the demonstration. According to the latter, we could hardly identify hypotheses with veritative indeterminacy.

Thus, our paper will focus on the inferential use of the notion of hypothesis. And from this, we will show that, from the texts of the Greeks themselves (especially Plato and Aristotle), that sense A pointed out by Lalande does

2 "I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction." (Newton, 1726/1999, p. 943).

not exhaust all its prominence in logical proofs. We will then develop this idea.

3. Proposition *vs* use

As we pointed out above, the tripartition presented by Lalande seems to suggest the standard meanings of the notion of hypothesis, as reflected in the texts today. However, there is a complementary sense that seems to have been lost and that we would like to highlight, which is subsidiary to the connotation A. As we indicated before, this sense was already present in the first Greek geometers and also recovered by Plato for the arguments made by Socrates in his dialogues. Plato points out that there is a type of examination 'by means of a hypothesis' (εξ ὑποθεσεως αυτο σκοπεισθαι), which comes from the geometers, and considers it adequate for making proofs in philosophy, as Socrates himself says in the Menon (86b-e):

> So it seems we are to consider what sort of thing it is of which we do not yet know what it is! Well, the least you can do is to relax just a little of your authority, and allow the question —whether virtue comes by teaching or some other way— to be examined by means of hypothesis. I mean by hypothesis what the geometricians often do in dealing with a question put to them; For example, whether a certain area is capable of being inscribed as a triangular space in a given circle: they reply — "I cannot yet tell whether it has that capability; but I think, if I may put it so, that I have a certain helpful hypothesis for the problem, and it is as follows: If this area is such that when you apply it to the given line of the circle you find it falls short by a space similar to that which you have just applied, then I take it you have one consequence, and if it is impossible for it to fall so, then some other. [...]" In the same way with regard to our question about virtue, since we don't know either what it is or what kind of thing it may be, we had best make use of a hypothesis in considering whether it can be taught or not [...] (Plato, 1952, p. 325)

At the end of the quote, the problem in question is outlined: (1) *Is virtue teachable or not?* Socrates borrows the demonstration method from geometers[3] and hypothetically will consider (2) *whether it is a science or not*. In other words, like the ancient geometers, Socrates is prevented from proving (1) directly and therefore considers the hypothesis that 'if virtue is a science,' then it would be proven that 'it is teachable.' On the same topic,

3 Is this also why Plato said for his Academy, 'Let no one ignorant of Geometry enter here'?

Aristotle offers more insights regarding this type of demonstration. Indeed, let's consider the following statements by Aristotle (41a39 and 50a19-24, respectively):

> And the same holds for all other arguments from a hypothesis, for in all of them the syllogism is for the substituted proposition, while the initial thesis is reached through an agreement or some other kind of hypothesis. (Aristotle 2009, 38)

> For example, if one had assumed the hypothesis that if there is not a single power of contraries, there is also not a single knowledge, and then one went on to argue that not every power is a power of contraries: not, for example, of the healthy and the unhealthy, for then the same thing would be healthy and unhealthy at the same time. Now that there is not a single power for all contraries has been proved, but that there is not a single knowledge has not been shown. And yet it is necessary to agree to this – though not on the basis of a syllogism, but on the basis of a hypothesis. (Aristotle 2009, 59-60)

We would like to highlight here three concepts that enrich sense A mentioned above by Lalande: (i) *agreement*, (ii) *substitution*, and (iii) *relation between two proofs*. Regarding the first, agreement, the dialogical character of the notion of hypothesis for Aristotle is remarkable. In other words, there are only hypotheses in the context of an *agreement* with an interlocutor. The latter is crucial and requires, from our perspective, a pragmatic approach for a suitable reading of the procedure, including —of course— *agents* and *contexts*. It is also significant to us that the Stagirite resorts to the idea of *substitution* (or, we dare to say, *surrogacy*), to think about the committed relation between (1) and (2) above. Indeed, Plato's example - which Aristotle must have known very well - tells us that A 'logically' substitutes for B in the sense that B is proved (by agreement) by proving A. The latter makes it clear that we are considering two proofs. We will go into more detail on the latter in the next point.

But first, a few comments: regarding the idea of an *agreement* between interlocutors, we must remember that it cannot be generalized for all of Aristotle's uses of the notion of hypothesis. Indeed, if we search for a single definition of hypotheses in their texts, we will immediately find different meanings to take into account that do not coincide with each other. For example, in *Posterior Analytics*, Book I Chapter 2 (Aristotle, 2002, p. 4), Aristotle distinguishes between two types of '*position acts*' (we follow

Crubellier's French translation of θέσις, '*actes de position*'): (i) *definitions* (which only say what an object is) and (ii) *hypotheses* (which say that something is or is not the case). But, on the other hand —in the sense that we seek to rescue— in Book I, Chapter 10 (Aristotle, 2002, p. 16), Aristotle distinguishes between things that are '*proven*' and those that are '*put*.' But the distinction it makes within this category is not the same as in chapter 2. Here he makes a *dialogical* distinction between:

i. things that are '*put*' in agreement with the interlocutor (he calls them *hypotheses in the strict sense*), and
ii. the things that the interrogator asks the interlocutor to *accept* (these are the *postulates*, Euclid retains this use in the *Elements*).[4]

Although these definitions do not coincide, they do not lead to dismissing the sense of agreement that we intend to highlight and which we consider to be of the most significant benefit for the study of demonstrations in logic.

Regarding the three concepts of agreement, substitution, and two proofs, W. D. Ross (1957) confirms what we said:

> The general nature of such proof is that, desiring to prove a certain proposition, we first extract from our opponent the admission that if a certain other proposition can be proved, the original proposition follows, and then we proceed to prove the proposed proposition (*to metalambanomenon*, 41a39). The proposed proposition is said to be proved syllogistically, the other not syllogistically but ex hypozéseos. (Ross, 1957, p. 371)

In other words, to summarize what we consider useful for our work, from our point of view, these three integrated notions move away from the identification of hypotheses with the antecedent of a demonstration. Indeed, we believe that an approach that integrates all three is required. Let us look at this in more detail below.

4. Generation of hypotheses as *performative acts*

In this section, we propose a *pragmatic* perspective of the concept of hypothesis based on our interpretation of Aristotle's quotations (41a39, 50a19-24). To do this, we distance ourselves from the identification between hypothesis and antecedent of a conditional – which belongs to what we will call a *static* approach, and we propose a *dynamic* concept of hypothesis based on notions of *use* and contexts.

4 We thank Prof. Michel Crubellier (Université de Lille, France) for these clarifications.

Emphasizing the notion of *use*, we propose that hypotheses result from a *performative act*. As in the three senses recovered by Lalande (see above), we join the idea that the introduction of hypotheses in scientific practice accompanies the use of conditionals. But we will oppose identifying hypotheses with the antecedent of the conditional. In this sense, we argue, the generated conditional results from the performative act of *hypothesizing*.

This act corresponds to the agreement reached by agents and points to an inferential commitment where there is a substitution relationship that compromises two proofs.

We should understand hypotheses as performative acts from John L. Austin's (1975) approach and not from that of Nicholas Rescher (2007), which distinguishes them from the 'inferential' ones.[5] In fact, following the perspective of Austin (1975), we argue that hypothesizing is the action of generating a particular type of relationship between parts (logical elements). In this way, we limit *hypotheses* to the field of dialectical interactions in which human beings and their actions as argumentative agents with purposes and goals play a central role. In this field, some considerations point to arguments' *context*, *possibility*, and *refutability* (*defeasible reasoning*), among other things.[6]
In short, these would be agents in interaction that use certain components to establish a particular type of *relationship* between them. This action is what we call 'generating hypotheses' or, in performative terms, *hypothesizing*. Now, we need to answer two important questions to complete our proposal: what kind of conditional relation are we talking about here? And what would be the *components* of this relationship?

5. Hypotheses as conditional *syntheses*

Regarding the first question, we argue that from the interactive agreement between two agents (two scientists), a hypothesis emerges in the form of a *conditional*. We claim that, on the one hand, there is an *agreement* between agents based on the action of *hypothesizing*[7]. And on the other hand, this agreement between interlocutors constitutes an inferential relation

5 Indeed, Rescher separates the performative type conditionals from the inferential conditionals kind (2007, p. 13). In the latter type, the conditional has implicational consequences: 'If you are in Valparaíso, you are in Chile.' The first type involves orders, promises, intentions, etc., such as 'If you do the work, I will pay you.' But in our approach, the underlying idea is -at least with inferential conditionals- that such a difference does not exist: a hypothesis is generated through a performative act. And what is this act? It is the performative act of agreeing on a relationship between agents.

6 We believe that Aristotle would agree with this because he places demonstrations by hypothesis among dialectics (*Topics* 100a18; *APr.* 24a20).

7 But we certainly do not argue the opposite —that behind every conditional, there is an agreement— which seems more doubtful and is not the objective of this article.

(as Aristotle already pointed out when describing it as *dialectical syllogism*), and this inferential relation —the conditional one— is called 'hypothesis.' Finally, to underline the pragmatic character of this agreement, we claim that the hypothesis generated is a logical synthesis of elements in the form of a conditional. By logical synthesis, we mean that there is a logical relation between the elements that the conditional relates. For this reason, we will call the hypothesis a *conditional synthesis*. In this sense, we argue that agents logically bring together elements in a conditional which are recognized in the general literature as the antecedent and the consequent. More details on these components will be provided below.

Some clarifications regarding the latter:

> 1. What is the new meaning of "hypothesis" as a conditional synthesis and not as the antecedent of a conditional? The new definition is that represented by the three notions of agreement, substitution, and two proofs mentioned above. In other words, hypothesizing —in agreement with an interlocutor— establishes an inferential relation[8] between logical elements, and such a relation corresponds to a conditional (antecedent and consequent) and which, from our point of view (suggested by A), in a specific case of hypothesis generation to be defined below, corresponds to a type of substitution.
>
> 2. As a background to our view on hypotheses, it is worth noting the work of Adams and especially Dorothy Edington (1986). She considers that 'conditional sentences (indicative)' do not purport to state a fact but to express an epistemic attitude. This point of view, against a propositional interpretation of the conditionals, looks for "a stronger-than-truth-functional "connection" between antecedent and consequent." (p. 3)
>
> 3. From the above, a question arises that is not present in the static approach. In the latter, the hypotheses would be the antecedent of these conditionals. But in our approach, we must clarify the following question: *What is the relation between conditionals and hypotheses?* One possible way to answer this question is as follows: the notion of conditional, already problematic in itself, imposes specific characteristics on the parts that make it up. The most important —and most accepted— is that the first part must be a condition

8 We speak of *inferential relation* as understood by Rescher: "[...] all types of conditionals can be understood and accounted for in terms of logico-conceptual derivability, so that deductive inference (⊢) constitutes the basis of conditionality (⟹) in general." (Rescher, 2007, p. 217).

for the second —antecedent and consequent. As noted above, we will consider this condition to be inferential. That is, *inferential* in the sense that for a *modus ponens*, the premises are the antecedent and the conclusion the consequent. But certainly, not all conditionals have deductive and monotonic relations between their parts. As is the case in the conditional 'If he comes, I leave' as opposed to 'If he comes, then 2+2=4.' In the first case —and not in the second— the consequent *one follows* the antecedent, so the structure 'If..., then...' is not enough to recognize a conditional. In this sense, we believe that this feature of conditionals is necessary for our hypothesis proposal but not sufficient. We will give more details below.

4. In summary, —answering the question at the beginning— the relation between conditional and hypothesis that we consider useful for this paper is as follows: *all scientific hypotheses are conditionals, but not all conditionals are hypotheses*. For example, the expression 'I will carry an umbrella in case it rains today' is a conditional expression, but we could hardly qualify it as a scientific hypothesis. Likewise, it is not the case that 'if the speed of galaxies hasn't slowed down, then dark matter doesn't exist,' which is both. We mean that the context of its utterance converts or allows us to identify a conditional as a hypothesis. That is, it is about the *use* being made of that conditional. It would then be those conditionals that establish inferential relations (hypotheses) that, firstly, have been formulated to solve scientific problems and, secondly, that if proved, increase our wealth of knowledge. However, we do not believe we can establish a strict demarcation criterion.[9]

6. The logical components of a hypothesis

Hypotheses, as conditional relations[10], in our view, can have at least two types of components: *propositional functions* and *proofs*. Thus, we will only analyze these two options: hypothesis as the conditional synthesis between propositional functions and conditional synthesis between propositions. For an analysis of other types of conditionals and their components, see —among others— Rescher (2007) and Sundholm (2019; 2012).

9 Indeed, demarcation is difficult because some conditionals that were part of the literature were later considered part of scientific knowledge. But the same would say a defender of the static approach, that many of his current hypotheses (such as antecedents of a conditional) were previously part of a fictional story.

10 These are conditionals and not material conditionals governed by truth tables (veritative-functional).

6.1 Hypotheses as Conditional Synthesis Between Propositional Functions

This type of hypothesis is composed of propositional functions and proposes the following: *that the elements that satisfy the antecedent must also satisfy the consequent*. The general scheme of this type of hypothesis is the following: *If x is P, then x is Q*. Consider the following examples:

a) If x is an isosceles triangle, then x satisfies that the angles of its base are homologous.
b) If $x \leq 3$, then $x \leq 4$.
c) If he comes, I'm leaving.

They all have in common that it is stated that if an element fulfills the first condition (the antecedent), then it must fulfill the second (the consequent). It is not stating that it fulfills both because, in that case, we should express it as a conjunction. In other words, there is the same committed element for *a* and *b*, which is not true for *c*. Indeed, in the latter case, it would be a conditional that does not meet the characteristics we have indicated for this type of conditional, which we call a hypothesis.

How do we evaluate such a hypothesis? It is certainly not a formulation intended to be tested against phenomena in the framework of a scientific practice of testing. In the case of the *abc* triangle, for example, it can be tested —in geometry— by following an axiomatic procedure. In these cases, it is clear that to generate a hypothesis is to generate this conditional relationship between the parts, where the consequent logically depends on the antecedent. The initial non-exhaustive criterion that we propose to differentiate conditionals from those that are also hypotheses is, for the latter, that of the same element satisfying two propositional functions at the same time.

In short, a hypothesis is the result of an interactive agreement between agents that generates a conditional synthesis between propositional functions.

It is important to note that our approach to hypotheses as conditional syntheses also reaches Lalande's interpretation of senses B and C. For example, in the following case: 'If *x* is heavier than air, *x* falls freely'. But in this case, the sustainability of the hypothesis is given by the possibility of contrasting it with the phenomena, i.e. submitting it to evaluation within the framework of a scientific practice. A logical relation is not announced in it as it is established in the previous ones and which characterizes this 'mathematical' sense of the hypotheses as it was already proposed by the first Greek geometricians. The novelty of our proposal is that we

distance ourselves from the static perspective that defines the hypothesis as the antecedent of each one. The static approach would tell us that in proposition *b* above, the hypothesis is '$x \leq 3$'. We believe that the latter does not make much sense. Even if we questioned them they would reply that it is a hypothesis because 'If $x \leq 3$, then $x \leq 4$', confirming our point that the hypothesis is the complete expression, i.e. the conditional. Indeed, we believe, they will have to resort to the full expression in order to identify '$x \leq 3$' as a hypothesis, since in other contexts it clearly is not. Therefore, in our view, it is necessary to differentiate between hypotheses and assumptions. We develop this further below. Let us now look at the second type.

6.2 Conditional synthesis between propositions or proofs

This second type points to a synthesis between propositions. But it should not be confused with the previous class because, in that class, there is a relation of satisfiability between propositional functions, while in this one, we talk about propositions (already saturated propositional functions). And what's the difference? We believe that the Greek geometers answered the latter clearly and that Plato and Aristotle later recovered it for our philosophical mill: is a relation between *justifications* or *evidence* of those propositions, and it is here that the notion of substitution acquires meaning. Let's consider the following example: when we hypothetically establish that 'if subject X was at the crime scene at the time of death, he is one of the suspects,' the consequent will be proven if we prove the former. But, to paraphrase Aristotle, the proof of the former (for example, looking at the camera that points to the only entrance/exit of the crime scene) is not the proof of the latter (since we would only be proving that he was at the crime scene). That is, the consequent would be proved by the hypothesis we established. That is because of the agreement we made with an interlocutor who agreed that if we proved the former, the latter would be proved. It is in this inferential framework that we say that the first statement substitutes the second. It should be borne in mind, as Aristotle points out in his *Posterior Analytics* (76b27-34), that in some cases of a demonstration by hypothesis, such as demonstrations by the impossible, this agreement is *demanded* of the interlocutor (see 'postulate' above).

Another example would be the conditional synthesis that 'if my cup of coffee has three tablespoons of sugar, then it also has two.' This hypothesis, from our perspective, states that if we prove the first (the antecedent), the second (the consequent) would be proved. Once again, we have two proofs in mind: the proof of the antecedent, on the one hand, and the proof of the consequent, on the other. The latter corresponds to the agreement we made with an interlocutor.

In short, the idea of a relation between proofs is complementary to a relation between propositions. In fact, in our view, we have (at least) two possibilities. A first one, where the relation is between at least two propositions such that the antecedent is the premise(s) and the consequent is the conclusion of an argument (deductive or ampliative).[11] And a second case, also between propositions, in which we recognize that the proof of the antecedent bears a relation of dependence with the proof of the consequent. In other words, in the latter case, the conditional is not properly between two propositions but between their proofs.

An interesting logical development of this idea of a relation between two proofs is found in the work of Per Martin-Löf specially in 1984. In effect, in the CTT approach created by Martin-Löf, the explanation of the meaning of the conditional AÉB (Martin-Löf 1984, pag. 7) consists on a method which takes any proof of A into a proof of B. So, for Martin-Löf, hypothetical judgements are "judgements which are made under assumptions" (idem p. 9). If we assume that A and B are propositions (it could be *sets* on Martin-Löf perspective), the generalized form of this judgements is:

$$b(x):A(x)\ (x:B)$$

Which is interpreted as follows: b(x) is a (dependent) proof object of A(*x*), *provided x* is a proof object of the proposition B.

In connection with Martin-Löf's work, Goran Sundholm interprets the conditional [(2) if A is true, then B is true] as follows:

> The conditional (2) is a hypothetical judgment in which hypothetical truth is ascribed to the proposition *B*. Its verification-object is a dependent proof object *B*:proof (*B*) [*X*:proof (*A*)], that is, b is a proof of *B* under the assumption (hypothesis, supposition) that *x* is a proof of *A*. (Sundholm, 2019, p. 555)[12]

From our approach, this dependence is established by agreement between the agents. That is, the dependence is established from the agreement with an interlocutor who concedes that if *x* is a proof of *A*, *b* will be the proof of *B*. Comparing it to the case of Menon, from our perspective, proof *x* that

11 In the sense in which Rescher (2007, p. 217) says: "[...] all types of conditionals can be understood and accounted for in terms of logical-conceptual derivability, so that deductive inference constitutes the basis of conditionality (⟹) in general."

12 The conditional (2) is a hypothetical judgment attributing a hypothetical truth to proposition B. Its verification object is a dependent proof object b:proof (B) [x:proof (A)], that is, b is a proof of B under the assumption (hypothesis, assumption) that x is a proof of A.

'virtue is a science' is proof b that 'virtue is teachable'. But they are two demonstrations: the *x* proof of *A* and the *b* proof of *B*, which is by hypothesis (Aristotle *dixit*).

Finally, regarding the type of evidence or justification considered in each type, we believe that the propositions we consider in this second type do not seem reducible to hypotheses of the first kind. Let's consider as an example the antecedent of the *Meno* hypothesis: 'if virtue is a science.' We believe that it is not reducible to 'if *x* is a virtue, *x* is a science'. In effect, in this last expression we would be proving that the different candidates that satisfy the function 'x is a virtue' will also satisfy at the same time that 'x is a science'. But this conditional does not seem to prove that 'x is a virtue' but rather that what is a virtue is also science. This idea leads us to the following reflection: for each conditional synthesis between propositional functions, a more basic concept allows us to propose such a synthesis. For example, reconsider the following cases similar to 'if *x* is a virtue, *x* is a science':

a. If $x \leq 3$, then $x \leq 4$.
b. If a magnet attracts *x*, *x* is iron.
c. If *x* is heavier than air, *x* falls.

For each of them, there is a fundamental concept that we are not proving, but we are considering it by stipulation. For example, in 'a', we are not proving the notion 'less than,' but on this conceptual background, we hypothesize that any candidate who correctly instantiates the antecedent instantiates the consequent. In 'b', we are not proving that 'magnetite attracts objects,' nor in 'c' the 'fall or not of massive bodies.' In both cases, the respective hypotheses conceptually presuppose this.

Therefore, we believe that from our perspective, it would not be correct to talk about a conditional of conditionals as a hypothesis: 'If [if *s*, then *t*], then [if *m*, then *n*].'

7. Hypothesis and assumption

We believe that our proposal clarifies and is complementary to that of Martin-Löf and Sundholm's developments. In our approach we insist that the hypothesis must be distinguished from the assumption. That way, we can prove a hypothesis without the assumption of losing its quality as such. Indeed, an important benefit of this distinction is that we can recognize when a propositional function or a proposition becomes an assumption: only when it is the antecedent of a hypothesis. For example, the proposition 'the time is 13:30' is an assumption only if it appears in the

hypothesis 'if the time is 13:30, we missed the morning train'. Otherwise, it is only a proposition. The same for '$x<3$': it is only an assumption in 'if $x<3$, then $x<5$'. Once we have proved both (if we succeed, of course), we understand that the hypothesis disappears, but we retain the assumptions in each expression. That is, an assumption is an expression that must be saturated or proven as an antecedent for the desired consequence to follow. In this way, we maintain the sense A pointed out by Lalande but under the denomination of assumption. In the same way, this distinction allows us to abandon the widely expanded idea that hypotheses or assumptions (used interchangeably) are such because (i) we do not know if they are V or F or (ii) because they are dubious but credible statements. If the latter were the case, it would turn all fictional literature into hypotheses.

To recapitulate, a hypothesis is a conditional synthesis, and its antecedent is an assumption. This assumption can be i) an unsatisfied propositional function or ii) an unproven or unjustified proposition. Apart from this use, we believe it makes no sense to talk about assumptions. For example, nobody would read Verne's story *From Earth to Moon* to their children regarding assumptions and hypotheses. Nevertheless, this does not prevent much of the story content from being used later as antecedents of hypotheses, in a well-determined scientific practice.

8. Some applications

In general terms, a pragmatic and dynamic approach to hypotheses as a conditional synthesis is applicable even where the static approach recognizes a hypothesis but reduces it only to the antecedent of a conditional. To exemplify, we will briefly focus on an emblematic case: *abduction*.

In fact, —for abduction— following the general scheme that postulates a hypothesis to explain a surprising case, our proposal postulates that the relationship between the surprising case and the hypothesis responds to a conditional synthesis, the result of an interactive agreement between the agents of argumentation. The novelty we introduce is to call as hypothesis the introduced conditional and not the antecedent of the conditional, as does the static approach. That is, following C.S. Peirce (PC 5.189):

> The surprising fact C is observed. But if A were true, C would be a matter of course.
> Hence there is reason to suspect that A is true.

And for the static perspective: an explanation or hypothesis A for the surprising case *C*. From our perspective, the hypothesis is presented with the conditional 'if *A*, then *C*' (interactive agreement), where *A* is

the assumption. Our proposal is close to what Woods calls 'subjunctive conditional connective' which they denote by the sign ⇝ (Woods, 2013, p. 369). Hypothesis A is thus connected to *C* by means of this logical operator: $A \rightsquigarrow C$. But in our proposal, for a modal reading of abduction, such as the one proposed by these authors, we would prefer to accompany the conditional synthesis with a modal operator of the kind [Abd]. Then, the above expression would look like '[Abd] (if *A*, then *C*).'

Finally, we will give a case within the framework of the ludic semantics of *Dialogics* to show that in this pragmatic framework, our idea of agreement that bases the generation of a hypothesis is already latent. To this end, let's consider the *particle rule of the conditional* in first-order semantics in *Dialogics* (Redmond & Fontaine, 2011; Rahman & et al., 2018; Clerbout & McConaughey, 2022). The rule is as follows:

	Assertion	Challenge	Defense
$\rightarrow$	X-!-$A \rightarrow B$[13]	Y-!-A	X-!-B

Our point is that the challenge of Y is a concession that reveals an agreement between X and Y. It is based on this agreement that Y grants X the antecedent to prove the consequent. So we could say that Y gives him the antecedent as proven. And the other player will use this concession to prove the consequent. Let's look at the following case:

Y it operates as follows for the thesis $(p \text{ Ù } q) \rightarrow p$:

O			P		
				$(p \wedge q) \rightarrow p$	0
1	$p \wedge q$	0		p ☺	4
3	p		c	$?\wedge_{L}$	2

13 "!" and "?" are *force* symbols. In Dialogic, all tripartite expressions where the symbol "!" occur, correspond to a formula that must be defended.

The attack of O ($p \wedge q$) is a concession that makes O to P so that it can prove the consequent p. From our perspective, we understand that this concession is based on a previous agreement between O and P, that of having accepted that ($p \wedge q$) $\rightarrow p$ is a conditional. A consequence of this reading is that everything conditional in dialogic is the fruit of an agreement. In other words, in a way, dialogical would be presented as the ideal framework to capture our approach to hypotheses as interactive acts. The latter is a topic to be developed in a future paper.[14]

9. Conclusion

The objective of our paper was to defend a pragmatic and dynamic approach to the inferential generation of hypotheses in scientific practice. Taking a distance from perspectives that identify "hypothesis" with the antecedent of a conditional relation, we have proposed that hypothesis is the result of an interactive agreement between agents and that it is expressed as a conditional relation. In this way, we have identified the generation of hypotheses with the performative act of *hypothesizing* which corresponds to the action of generating a conditional relationship between *propositional functions* and *proofs*. We have built our proposal from the notion of 'demonstration by hypothesis' outlined in a general way by Aristotle in his *Prior Analytics*. From it, we have recovered the notions of (i) agreement, (ii) substitution, and (iii) relation between two proofs that have allowed us to limit hypotheses to the field of dialectical interactions, in which human beings and their actions as argumentative agents with purposes and aims play a central role.

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14 An extended version of these ideas and some more applications can be found at Redmond (2021a, 2021b); Redmond & López-Orellana (2022, 2023a, 2023b y 2023c); Redmond & López-Orellana & Paniagua (2021).

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